# ESTIMATION OF THE RATE OF DISPLACEMENT OF AIR IN A CONFINED SPACE WITH NATURAL CONVECTION

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The author examines a method of determining the rate of vertical displacement of air due to natural convection, using experimental data obtained by means of an interferometer. Design formulas are given.

The difficulties associated with the quantitative evaluation of the rate of convective motion of liquids and gases have been duly emphasized in various investigations [1]. The rate $v_y$ of vertical displacement of air in a confined space due to internal heat sources, without flow due to forced convection, can be estimated from the experimental results in the form of some empirical vertical temperature distribution. For this purpose we shall use the data of tests on the model with internal heat sources described in [2].

The vertical motion of an element of mass $\Delta m$ is due to the temperature difference (or, which amounts to the same thing, density difference) between the heated air and the surrounding medium $\Delta T = T - T_0$. Then the product of $\Delta m$ and the acceleration $d^2y/d\tau^2$ will equal the buoyancy force $\Delta m g \beta \Delta T$, i.e.,

$$\Delta m \frac{d^2 y}{d\tau^2} = \Delta m g \frac{T - T_0}{T_0}$$

or

$$\frac{dv_y}{d\tau} = g \frac{T - T_0}{T_0}.$$

The element $\Delta m$ does not remain constant in its upward motion, since it mixes with the surrounding medium; in fact, there is an inflow of entrained air particles. The modified volume of heated air will continue to rise after mixing, but at a different velocity. The temperature T of the element of volume will also change.

Putting the law of variation of temperature with height and time in the form of some function $T = f(h, \tau) = f(y, \tau)$, we may write

$$dv = \frac{g}{T_0} \int_{\tau=0}^{\tau} [T(y, \tau) - T_0]\, d\tau.$$

It is very difficult to obtain an analytic expression $T = f(y, \tau)$ for the unsteady regime of convective heat release in a small limited space. Studies of the steady temperature field have been made, however, by means of an optical instrument – the interferometer – and quantitative data are available that give the temperature T in terms of the geometric parameters of the space, and the temperatures $T_s$ of the sources and $T_w$ of the enclosing walls, and also permit the heat source power $\zeta = N_s/N_{max}$ to be taken into account.

For a two-dimensional space, the heat transfer process is described by the Fourier-Kirchhoff equation

$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = a \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right). \tag{1}$$

Taking the origin of the xyz coordinates at the center of symmetry of a circular heat source, the vertical variation of air temperature may be represented by the expression [2]:

$$\theta = A \zeta^m \eta^n \tag{2}$$

or

$$T = T_w + A \zeta^m (T_s - T_w) \left( \frac{h_i - h_s}{H_M - h_s} \right)^n. \tag{2'}$$

In (1) and (2) there are three unknowns: $v_x$, $v_y$ and T. We also require the equation of motion:

$$v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} = g \beta (T - T_0) - \frac{1}{\rho} \frac{\partial p}{dy}. \tag{3}$$

Equations (1)-(3) will be valid throughout the space, if we exclude the boundary layer at the heat source surface, and the thermal boundary layer at the walls.

From (2') we can find the values of the first and second derivatives $\partial T/\partial y$ and $\partial^2 T/\partial y^2$ for the vertical section in question. The derivatives $\partial T/\partial x$ and $\partial^2 T/\partial x^2$ remain unknown.

On the basis of the results of experimental research [2], as a first approximation we may assume the law of variation of temperature along the x axis to be $T_x$ = const. Then, with $\partial T/\partial x = 0$, the first two equations take the form:

$$v_y \frac{\partial T}{\partial y} = a \frac{\partial^2 T}{\partial y^2}; \qquad (1')$$

$$T = T_W + A\zeta^m (T_s - T_W) \left(\frac{h_i - h_s}{H_M - h_s}\right)^n. \qquad (2')$$

Introducing the relations $h_i - h_s = y$ and $A\zeta^m[T_s - T_W/(H_M - h_s)^n] = A_0$, may be written as:

$$T = T_W + A_0 y^n. \qquad (2'')$$

Inserting in (1') the values of the derivatives $\partial T/\partial y$ and $\partial^2 T/\partial y^2$ from (2"), we have, after a simple transformation,

$$v_y = a(n-1) y^{-1}$$

or

$$v_y = a(n-1)/(h_i - h_s). \qquad (4)$$

Determining the value of the parameter n from (2'), we can finally write

$$v_y = a\left[\frac{\lg\theta - \lg(A\zeta^m)}{\lg\eta} - 1\right] \frac{1}{h_i - h_s}. \qquad (5)$$

When temperature gradients are present in a direction parallel to the x axis, Eqs. (1)-(3) must be supplemented by the relation $T_x = f(x, \ldots)$; from their joint solution, the value of $v_x$ may be obtained.

Analysis of the experimental results, and comparison with the curve of (5), indicates that the latter is valid within the limits mentioned, excluding the source boundary layer region and the boundary layer of the surrounding walls.

## NOTATION

$\tau$ – time; x, y – coordinates of point; $T$, $T_0$, $T_W$, $T_s$ – temperature of air at point investigated, mean temperature of surrounding medium, temperature of inside face of wall, temperature of surface of heat source; $v_x$, $v_y$ – horizontal and vertical components of velocity of air; a – thermal diffusivity; $\theta = (T - T_W)/(T_s - T_W)$ – relative temperature; $\eta = (h_i - h_s)/(H_M - h_s)$ – relative height of point; $\zeta = N_i/N_{max}$ – relative power of heat sources; A, m, n – experimentally determined coefficients.

## REFERENCES


1. M. A. Mikheev, Fundamentals of Heat Transfer [in Russian], Gosenergoizdat, 1949.
2. L. T. Bykov and V. V. Malozemov, IFZh, no. 2, 1965.


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